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Prompt: FOILing Understanding

Many algebra teachers offer the mnemonic FOIL (First Outer Inner Last) as a means of helping students learn and remember how to multiple binomials. While this is popular and appealing, can carry the unintended message that mathematics learning is based on remembering rules to apply in a particular situation and misses the opportunity reinforce that mathematics is based on logic and reasoning.

Mathematical Focus 1: understanding polynomial multiplication.

Using the following application of the distributive property:
$3x(x + 2) = (3x)x + (3x)2$ or $(x + 2)3x = x(3x) + 2(3x)$
to motivate the multiplication of two binomials:
$(3x + 1)(x + 2) = (3x + 1)x + (3x + 1)2 = (3x)x + (1)x + (3x)2 + (1)2$
in at manner that can be generalized to all polynomial multiplications:
$(x^2 + 3x + 1)(x + 2)$ and beyond

Mathematics Focus 2: polynomial multiplication as generalized arithmetic.

Using two-digit arithmetic multiplication:
$\begin{array}{r} 31 \\ \times\ \underline{12} \end{array}$ or 31 x 12 =

as tool for binomial multiplication:
$(3x + 1)(x + 2) =$

Prompt: Canceling Out Meaning

Often students and teachers refer to simplifying an expression by removing terms that sum to zero or whose product is 1 as "canceling." While the language is convenient and describes the end result of the term or factor "disappearing," it masks the true mathematical content and can be misinterpreted or misapplied by students in other instances.

Mathematical Focus 1: Emphasis on additive or multiplicative identity by using terms like "zero-sum" or "multiplicative identity" when solving equations.

Solve: $3x – 2 = 4 - 5x$
"start by adding $5x$ to both sides" $3x – 2 + 5x = 4 – 5x + 5x$ ("the $5x$'s cancel" vs "$-5x + 5x$ sums to zero)
$8x – 2 = 4$
$8x = 6$

“divide by 8” $\frac{8x}{8}=\frac{6}{8}$ (“the 8’s cancel” vs “8 divided by 8 is 1”)

$x=\frac{2\cdot 3}{2\cdot 4}=\frac{3}{4}$

Mathematical Focus 2: Emphasis on additive or multiplicative identity by using terms like “zero-sum” or “multiplicative identity” when simplying expressions.

Prompt: Decimal Representation involving Infinity

In a first-year algebra course early in the school year, the teacher is reviewing numerical concepts and mentions that 0.99999… = 1. Students challenge this idea: “No, 0.9999… can’t be equal to one, it gets closer and closer but doesn’t quite reach it the whole number 1. In fact, the correct statement would be 0.9999… < 1.”

Mathematical Focus 1: the limit process
Similar to asymptotes in graphs, what does the ellipsis (…) mean? From an epsilon-delta perspective, how can the limiting process of “arbitrarily close” justify the equal sign in the expression?

Mathematics Focus 2: the concept of infinity
How do finite beings understand the concept of going on without end. This overlaps with the limit process but can be approached through the infinite series: .9 + .09 + .009 + …
One-to-one correspondence as a basis for countably infinite

Mathematics Focus 3: proof
Proving the equality using an informal or rigorous method.
1/3 = 0.3333…. and multiplying equation by 3 yields 1 = 0.99999….
Or
Let N = 0.99999…..
Then 10N = 9.999….
After subtracting latter from former:
9N = 9 which implies N = 1
Or
With Dedekind cuts or Cauchy sequences at higher levels.

Prompt: Graphical representations of real and complex solutions to quadratic equations.

Roots of quadratic equations can easily be understood to be represented as the x-intercept of the quadratic functions graph. Can the complex roots be represented as an intercept?

Mathematical Focus 1: representing real-number solutions to quadratic equations graphically. Connecting the discriminant from the quadratics formula to the number of solutions of a quadratic equation to the number of x-intercepts of the graph of a quadratic function.
Mathematical Focus 2: representing complex numbers graphically and representing the complex-number solutions to the quadratic equations graphically ("Connecting Complex Roots to a Parabola's Graph" in ON-Math , Spring 2003: http://my.nctm.org/eresources/view_article.asp?article_id=6168)

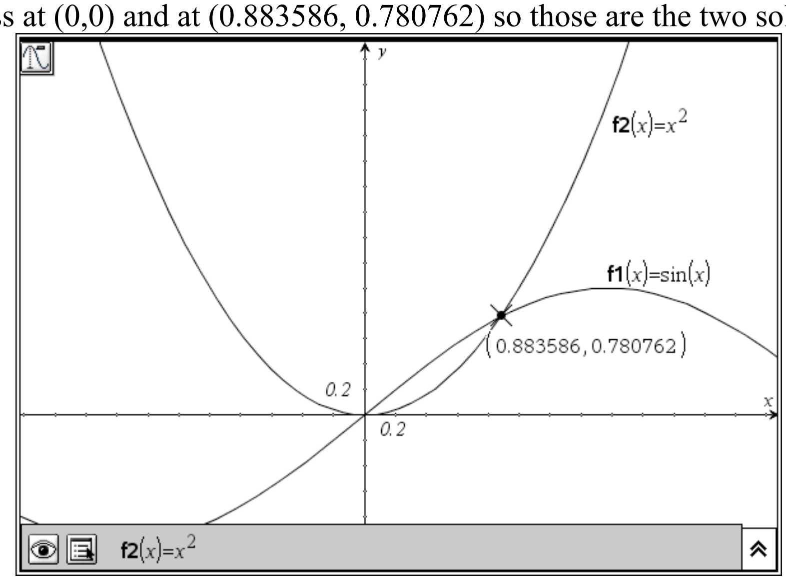
Prompt: Ratinonalizing the Denominator

The $\sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$. No, no, the correct answer is $\frac{\sqrt{2}}{2}$. You must ALWAYS rationalize the denominator. What motivates rationalizing the denominator? When might it be appropriate and when might it not matter at all?

Mathematical focus 1: conventions
Mathematics focus 2: reasons for rationalizing denominator.

Prompt: Using graphical methods to solve equations.

We can solve equations graphically. For example, to solve the equation $\sin(x) = x^2$, we can graph the left-side, $y = \sin(x)$ and the right-side, $y = x^2$ and find where they cross. See, they cross at (0,0) and at (0.883586, 0.780762) so those are the two solutions.



Mathematical focus 1: what does it mean to solve an equation
Mathematical focus 2: understanding and translating between solutions in symbolic vs graphical representations.

Prompt: Principal square roots

$\sqrt{9} = \pm 3$ as a misunderstanding of the radical symbol representing the positive or principal root.

Mathematical focus 1: conventions and definitions
Mathematical focus 2: definition of function (univalent)
Mathematical focus 3: rational and irrational exponents